(c) Entities are treated as if widely spaced in a fluid whose motion is otherwise steady but elsewhere it is accepted that every plane perpendicular to the y-axis, and therefore the entire fluid, consists of entities in motion.

There is a natural desire, perhaps felt particularly strongly with the turbulence problem, for simple, consistent, comprehensible physical models, even at some risk of over-simplification, but I have sometimes felt that the feasibility of such models is not sufficiently distinguished from their desirability. Physical concepts are most valuable for constructing a length of track on which to start in motion a train of thought. The train tends to become airborne during the mathematical development section and when, much further on, it comes down on a convenient stretch of track we are inclined to ignore the fact that it was ever off the ground, and has perhaps, being a well-behaved train, now adjusted to a different gauge. At the end of our travel we ought to judge the correctness of our journey by the merits of our new surroundings, yet we tend also to retain the feeling that an additional recommendation is that it has followed a continuous physical line. My view is that the usefulness of the theory proposed by Mr. Tyldesley and Professor Silver must rest entirely on the correctness of its predictions, and that rather more evidence is required than diffusivity ratio comparisons, which are in any case difficult to measure accurately. It is general experience that alternative, and widely dissimilar, models can all show some initial success in predicting observed trends, provided that the basic physical laws of the situation have not been completely violated. I would rather doubt whether there is much more to be gained from the statistical mechanics approach to turbulent flow, but I will nevertheless await with interest further developments of this particular theory.

V. WALKER

University of Bradford
Bradford 7
England

REFERENCE

1. J. R. TYLDESLEY and R. S. SILVER, The prediction of the transport properties of a turbulent fluid, *Int. J. Heat Mass Transfer*, **11**, 1325–1340 (1968).

P.S. It has been brought to my attention by Mr. P. Bradshaw that a further immediate test of the proposed theory can be made. It is well known that in a jet and in pipe or boundary-layer flow outside the viscous sublayer the eddy diffusivity is practically independent of viscosity, whereas according to the theory in the paper ε_μ is inversely proportional to μ. This may well be the most serious objection to the theory as it stands.

V.W.

Int. J. Heat Mass Transfer. Vol. 12, pp. 1209–1210. Pergamon Press 1969. Printed in Great Britain

NOTE ON THE PREDICTION OF THE TRANSPORT PROPERTIES OF A TURBULENT FLUID

(*Received* 4 *March* 1969)

TYLDESLEY and SILVER [1] have presented an interesting "rational description" of a turbulent fluid. Their model of coherent lumps of fluid, suddenly created, and surrounded by fluid having the properties of the mean flow, although it is hardly original, may lead to a number of useful qualitative conclusions. However when the quantitative analysis of [1] is applied to the turbulent flow of gas in a channel for example, there is a point at which the argument becomes fallacious. This arises from the failure of the treatment to evaluate the magnitudes of the length and velocity scales involved in the analysis, and it has at least three important repercussions.

Examination of the correlation coefficient of axial velocity fluctuations, R_y, as measured by Comte-Bellot [2], shows that it maintains a value greater than 0·5 for separations, y, of up to 0·1 of the channel half-width, h, over almost the whole of the flow passage (except for a region close to the wall). This indicates that an appropriate value for the radius of an "energy-containing entity" is $\langle R \rangle \sim 0{\cdot}1\ h$: moreover it is the energy-containing entities that make the most significant contribution to the transfer of momentum, which is described by the Reynolds stress in equation (13). That the authors [1] have overlooked this fact is shown by their comparison of equations (8) and (9), suggesting a correspondence between $\langle R^2 \rangle^{\frac{1}{2}}$ and the microscale δ. The microscale, being a measure of turbulent velocity gradients and related to the rate of energy dissipation by viscosity, is a function of Reynolds number, and is considerably smaller

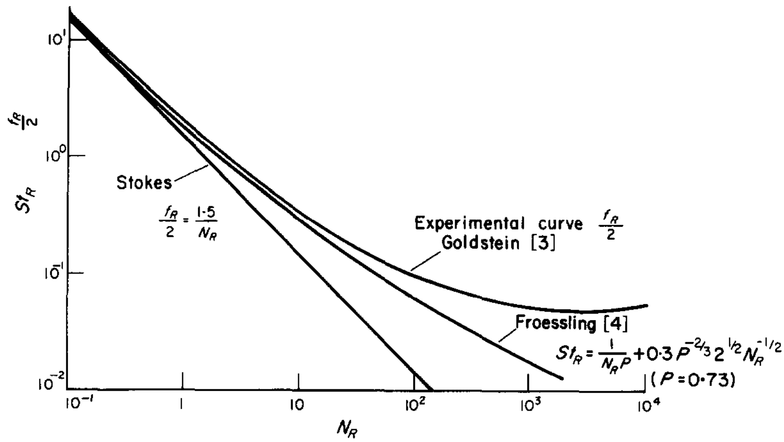

FIG. 1. Analogy between momentum and heat transfer for solid spheres in a fluid with $P = 0{\cdot}73$.

than 0·1 h at moderate channel Reynolds numbers. The order of magnitude estimate of R, together with the reasonable assumption that $V_0 \sim u_\tau$, where u_τ is the friction velocity for the channel wall, leads to the conclusion that when the channel Reynolds number $h\bar{U}/\nu$ is 10^5, the length parameter λ^* will be $\sim 10\,h$ and the entity Reynolds number $N_R \sim 400$. (From Comte-Bellot's [2] result that $\delta u_\tau/\nu \sim 100$, we deduce $\delta \sim 0{\cdot}025\,h$).

The most important repercussions of these estimates are:
(1) Due to the confines of the duct, the distance λ travelled by the entity since creation will always be $\ll \lambda^*$ and to a first approximation, equation (18) becomes:

$$\tau = + \frac{\rho\,\mathrm{d}\langle u\rangle}{\mathrm{d}y}\langle \lambda V_0\rangle,$$

the conventional mixing-length formulation, which will be only slightly modified when second order terms are included.
(2) As $\lambda \ll \lambda^*$, the assumption that the probability density of λ/λ^* is uniform between 0 and 1 is obviously incorrect. It is upon this assignment of probability density that the ratio of diffusivities calculated in [1] depends.
(3) Since $N_R \sim 400$, the use of Stokes' equation (1) for the loss of momentum by the entity during its flight, and of the corresponding equation (31) for the loss of heat is inappropriate. Defining friction and heat-transfer coefficients for the entity, the above equations may be written for the case of a sphere of radius R, in a manner which shows their analogous form,

$$\bar{F} = -\frac{f_R}{2}|(\bar{V} - \bar{v})|\,4\pi\rho R^2(\bar{V} - \bar{v}) = -6\pi\mu R(\bar{V} - \bar{v})$$

$$\dot{q} = St_R\,|(\bar{V} - \bar{v}|\,4\pi\rho C p R^2(T - T_f) = 4\pi K R(T - T_f).$$

Thus the authors [1] have put $f_R = (3/N_R)$ and $St_R = (1/N_R P)$. However when the friction coefficient data for solid spheres reviewed by Goldstein [3] is compared with the heat transfer correlation of Froessling [4] (Fig. 1), it is seen that the analogous behaviour breaks down for Reynolds numbers N_R greater than 100, and at that point, Stokes' friction coefficient is already in error by a factor of 7, due to inertial forces. It cannot be expected therefore that, even in an unrestricted turbulent flow, to which earlier objections might not apply, the turbulent Prandtl number will be correctly predicted by equation (47) of [1].

C. J. LAWN

Heat Transfer Section,
Central Electricity Generating Board
Berkeley Nuclear Laboratories
Berkeley, Gloucestershire
England

REFERENCES

1. J. R. TYLDESLEY and R. S. SILVER, The prediction of the transport properties of a turbulent fluid, *Int. J. Heat Mass Transfer* **11**, 1325–1340 (1968).
2. G. COMTE-BELLOT, Ecoulement turbulent entre deux parois paralleles, Publ. Sci. et. Tech. du Ministere de l'Air, No. 419 (1965).
3. S. GOLDSTEIN, *Modern Developments in Fluid Dynamics*, Vol. 2, p. 493, Oxford University Press, London (1938).
4. N. FROESSLING, *Beitr. Geophys.* **52**, 170 (1938).